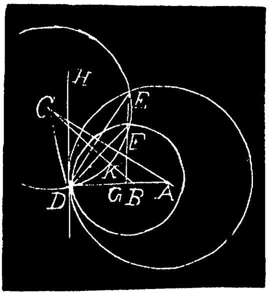
GEOMETRY.

80. Proposed by J. C. GREGG, Superintendent of Schools, Brazil, Ind.

One circle touches another internally, and a third circle whose radius is a mean proportional between their radii, passes through the point of contact. Prove that the other intersections of the third circle with the first two are in a line parallel to the common tangent of the first two. [From *Phillips and Fisher's Geometry.*]

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va., and the PROPOSER.



In the two triangles CDB and CDA, the angle CDG is common; also $DB : DC = DC : DA$.

$\therefore$ The two angles are similar and $\angle BCD = \angle CAD$, $\angle CBD = \angle ACD$. But $\angle DCB = \angle DEG$, both being measured by arc DK.

$\therefore$ $\angle DEG = \angle CAD$.

Since DE is perpendicular to CA, EG must be perpendicular to CA.

$\therefore$ HD and EG are parallel.

II. Solution by HENRY HEATON, M. Sc., Atlantic, Iowa, and J. SCHEFFER, A. M., Hagerstown, Md.

Taking the common tangent of the first two circles as the axis of x and the common diameter as the axis of y and supposing the tangent of the third circle, through the origin to make the angle α with the axis of x, the equations of the circles are,

$$x^2+y^2-2ay=0,$$

$$x^2+y^2-2by=0, \text{ and}$$

$$x^2+y^2-2c(x\sin\alpha+y\cos\alpha)=0.$$

If the first and third circles intersect,

$$y=0 \text{ or } \frac{2ac^2\sin^2\alpha}{a^2+2ac\cos\alpha+c^2}.$$

If the second and third circles intersect,

$$y=0 \text{ or } \frac{2bc^2\sin^2\alpha}{b^2+2bc\cos\alpha+c^2}.$$

If the line through the second intersections is parallel to the axis of x the two values of y are equal.

$\therefore$ $ab^2+ac^2=a^2b+bc^2$. Whence $c^2=ab$.

If $c=\sqrt{ab}$, the distance y of the intersections from the common tangent is in both cases

$$\frac{2ab\sin^2\alpha}{a+\sqrt{ab}\cos\alpha+ab}. \qquad \text{Q. E. D.}$$

Also solved by *COOPER D. SCHMITT* and *CHARLES C. CROSS.*